Applied Mechanics (27041) chapter-2

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Composition and Resolution of forces(বলের সংযোজন ও বিভাজন)

2.1 INTRODUCTION

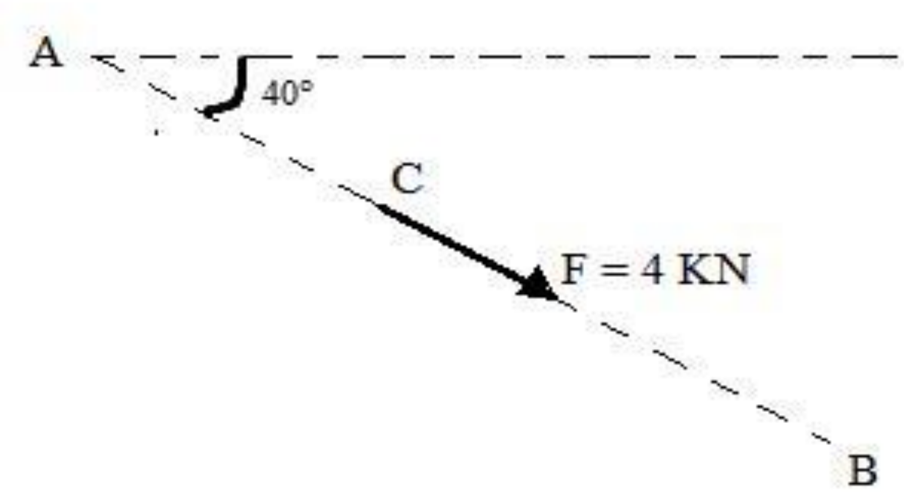
- Definition of force

‟ can be given in several ways. Most simply it can be defined as „the cause of change in the state of motion of a particle or body‟. It is of course, the product (multiplication) of mass of the particle and its acceleration. Force is the manifestation of action of one particle on the other. It is a vector quantity.

2.2 CHARACTERISTICS OF A FORCE

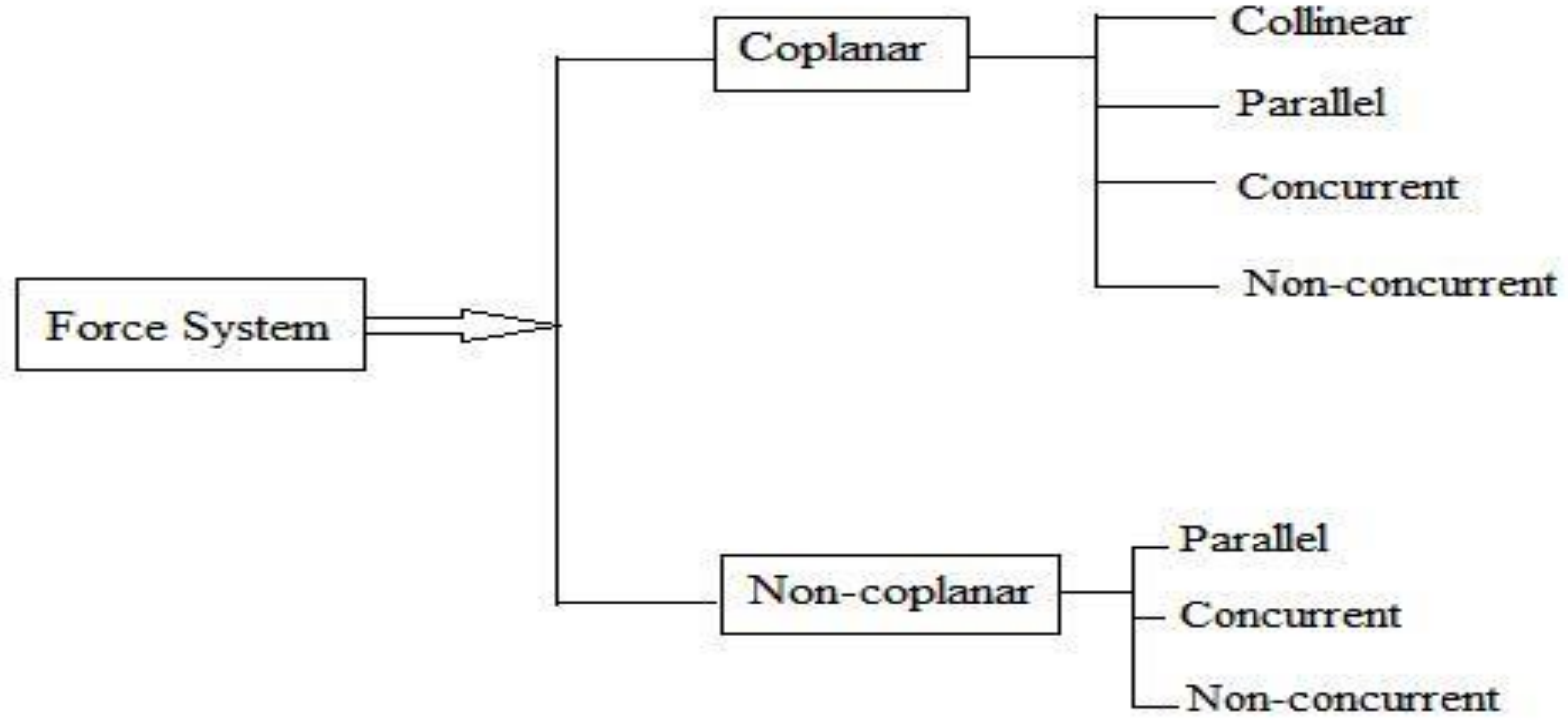
- A Force has following basic characteristics
- i) Magnitude
- ii) Direction
- Iii) Point of application
- iv) Line of action Force is represented as a vector
- .i.e an arrow with its magnitude

e.g. for the force shown in Fig. 2.1, magnitude of force is 4KN, direction is 40° with the horizontal in fourth quadrant, point of application is C and line of action is AB

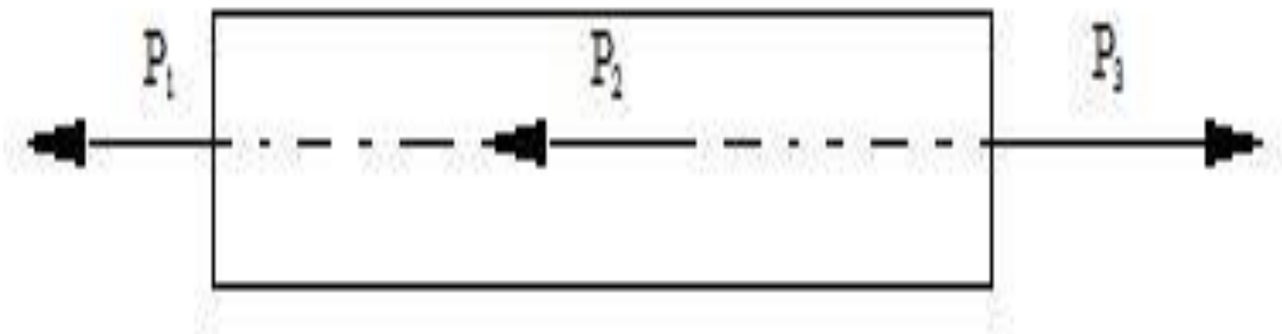


2.3 SYSTEMS OF FORCES

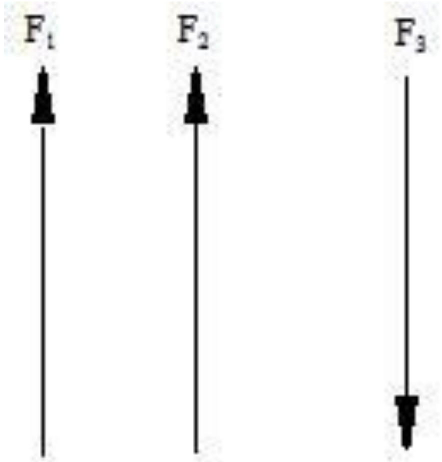
- When a mechanics problem or system has more than one force acting, it is known as a ‘force system’ or ‘system of force’.



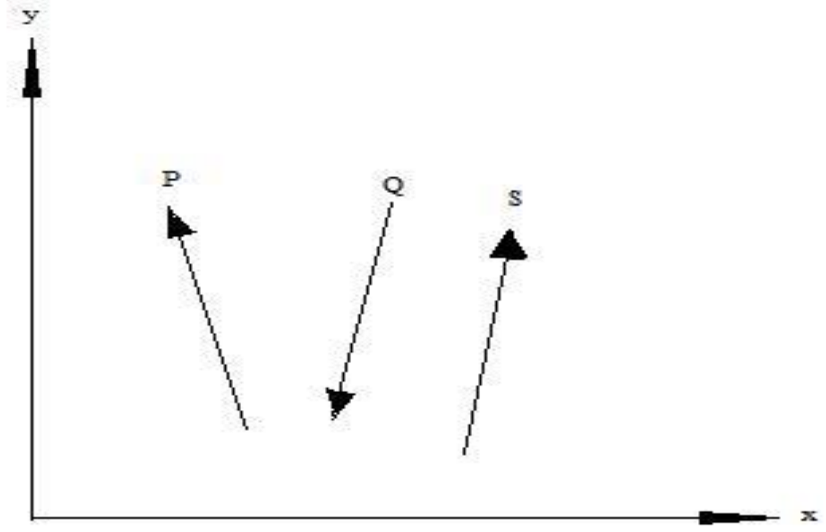
- **2.3.1 Collinear Force System**
- When the lines of action of all the forces of a system act along the same line, this force system is called collinear force system.



- **2.3.2 Parallel Forces**



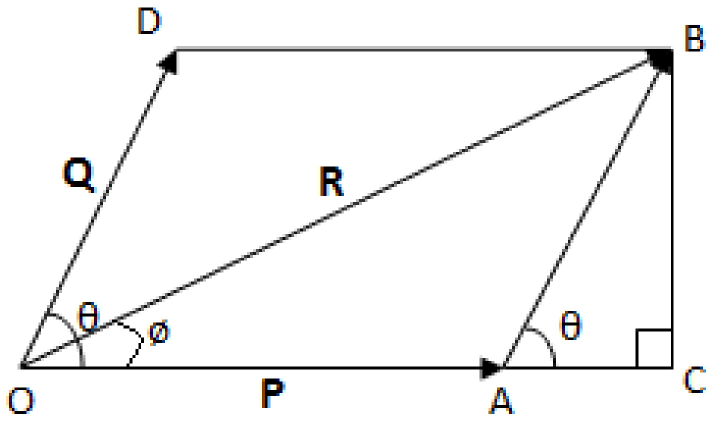
- **2.3.3 Coplanar Force System**
- When the lines of action of a set of forces lie in a single plane is called coplanar force system.
- **2.3.4 Non-Coplanar Force System**
- When the line of action of all the forces do not lie in one plane, is called Non-coplanar force system



Law of Parallelogram of Forces

This law states that two forces acting on a particle may be replaced by a single force (called resultant of the two forces) obtained by drawing the diagonal of a parallelogram whose two adjacent sides are equal to the given two forces. Let P and Q be two forces acting on a particle A as shown in Fig.4.11. Constructing a parallelogram ABCD taking P and Q as its adjacent sides, the diagonal AC gives the resultant force R of the two forces P and Q. The expressions for the magnitude and direction of R are obtained as follows: Considering the right angled triangle ACE, from the Pythagoras theorem,

- **ANALYTICAL METHOD:**
- Drop perpendicular BC to OA produced
  Now from the figure,
  $\angle AOD = \angle CAB$
- Consider the right angled triangle ACB
- Then, $CB = AB \sin \theta = Q \sin \theta$
  $AC = AB \cos \theta = P \cos \theta$
- Now from right angled triangle OCB
- $OB^2 = CB^2 + OC^2$
  $OB^2 = CB^2 + (OA + AC)^2$
  $OB^2 = CB^2 + OA^2 + AC^2 + 2 \times OA \times OC$
  $R^2 = Q^2 \sin^2 \theta + P^2 + Q^2 \cos^2 \theta + 2 \times P \times Q \times$
  $R^2 = P^2 + Q^2 (\sin^2 \theta + \cos^2 \theta) + 2.P.Q.\cos \theta$
  $R^2 = P^2 + Q^2 + 2.P.Q.\cos \theta$
  $\mathbf{R = \sqrt{(P^2 + Q^2 + 2.P.Q.\cos \theta)}}$



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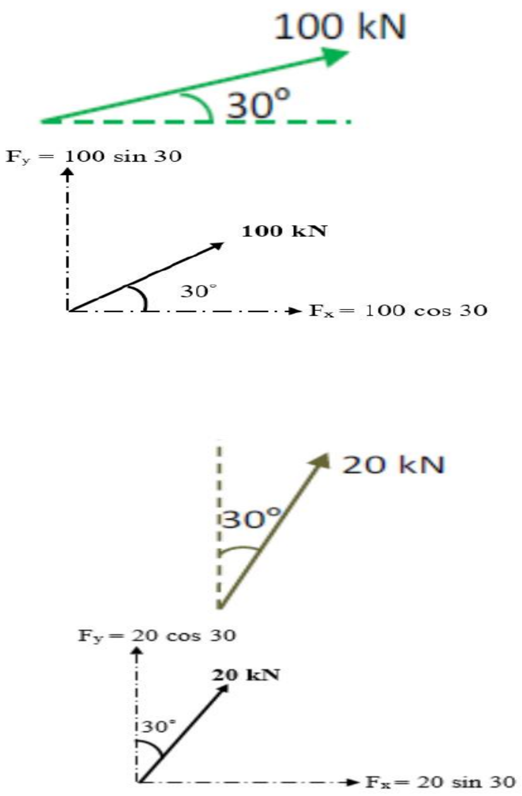
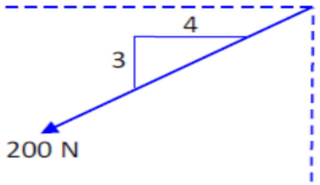
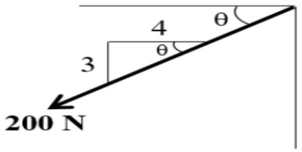
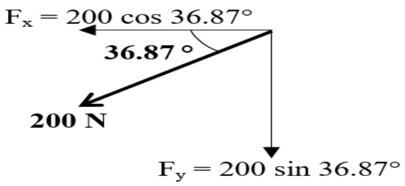
Resolve the given Force, determine X & Y components (Find Fx & Fy).

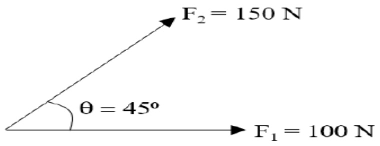


Determine □□□X & Y components (Fx & Fy).









Two forces of 100 N and 150 N are acting simultaneously at a point. Find the resultant if the angle between them is 45º

Solution

Consider $F_1 = 100$ N and $F_2 = 150$ N

Angle between Forces F_1 and F_2, $\theta = 45º$

Magnitude of Resultant, $R = \sqrt{F_1^2 + F_2^2 + 2\,F_1\,F_2 \cos\theta}$

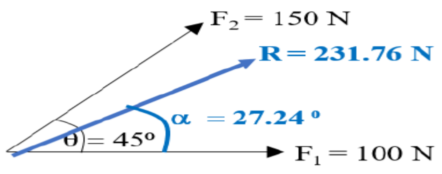
$$R = \sqrt{100^2 + 150^2 + (2 \times 100 \times 150 \times \cos 45)}$$

$$\mathbf{R = 231.76\ N}$$

Direction of Resultant (w.r.t. F_1), $\alpha = \tan^{-1}\left(\frac{F_2 \sin\theta}{F_2 \cos\theta + F_1}\right)$

$$\alpha = \tan^{-1}\left(\frac{150 \sin 45}{150 \cos 45 + 100}\right)$$

$$\boldsymbol{\alpha = 27.24º}$$



Solution

Consider F_1 = Smaller Force and F_2 = Bigger Force = 40 N

Angle between Forces F_1 and F_2 = θ = 120º

Angle of Resultant w.r.t. F_1 (Smaller Force) = α = 90º (as resultant is **perpendicular** to smaller force)

Direction of Resultant (w.r.t. F_1), $\alpha = \tan^{-1}\left(\frac{F_2 \sin\theta}{F_2 \cos\theta + F_1}\right)$

$$\tan\alpha = \left(\frac{40 \sin\theta}{40 \cos\theta + F_1}\right)$$

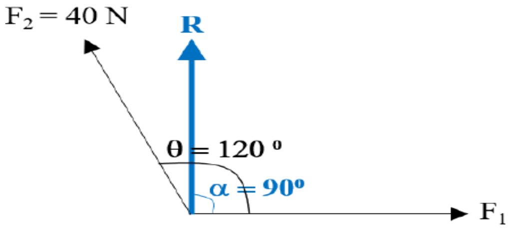
$$\mathbf{\tan 90} = \left(\frac{40 \sin 120}{40 \cos 120 + F_1}\right)$$

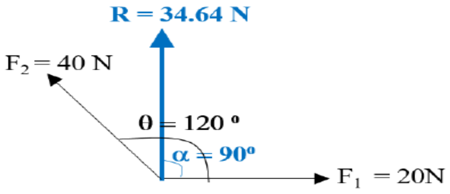
Subs. $\tan 90 = \frac{\sin 90}{\cos 90}$; $\qquad \frac{\sin 90}{\cos 90} = \left(\frac{34.64}{-20 + F_1}\right)$

Cross Multiplying, $\qquad \sin 90 \times (-20 + F_1) = \cos 90 \times (34.64)$

Subs. $\sin 90 = 1$ and $\cos 90 = 0$ $\qquad -20 + F_1 = 0$

or $\qquad F_1 = \mathbf{20N}$





Solution

Case 1: $\theta_1 = 90^0$ **and** $R_1 = \sqrt{10}$

From Parallelogram law of Forces, we have

Magnitude of Resultant, $R = \sqrt{F_1{}^2 + F_2{}^2 + 2\ F_1\ F_2 \cos\theta}$

$\sqrt{10} = \sqrt{F_1{}^2 + F_2{}^2 + (2 \times F_1 \times F_2 \times \cos 90)}$

Squaring both sides

$10 = F_1{}^2 + F_2{}^2$ => $\mathbf{F_1{}^2 + F_2{}^2 = 10}$ **.......(1)**

Case 2: $\theta_1 = 60^0$ **and** $R_1 = \sqrt{13}$

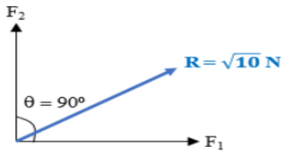
From Parallelogram law of Forces, we have

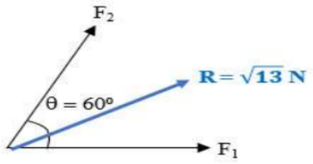
Magnitude of Resultant, $R = \sqrt{F_1{}^2 + F_2{}^2 + 2\ F_1\ F_2 \cos\theta}$

$\sqrt{13} = \sqrt{F_1{}^2 + F_2{}^2 + (2 \times F_1 \times F_2 \times \cos 60)}$

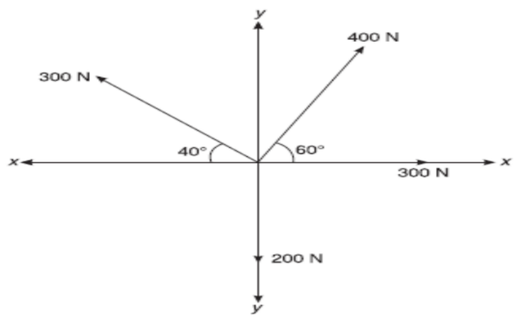
Squaring both sides

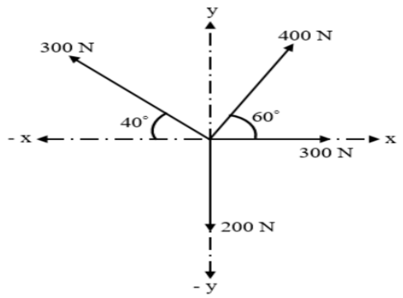
$13 = F_1{}^2 + F_2{}^2 + F_1F_2$ => $\mathbf{F_1{}^2 + F_2{}^2 + F_1F_2 = 13}$ **.......(2)**

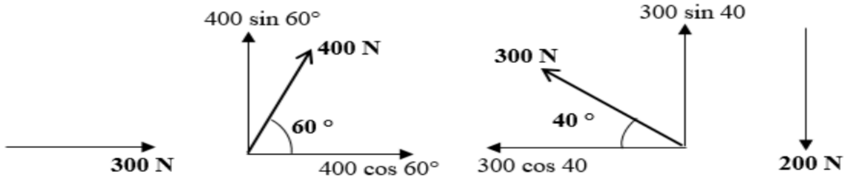




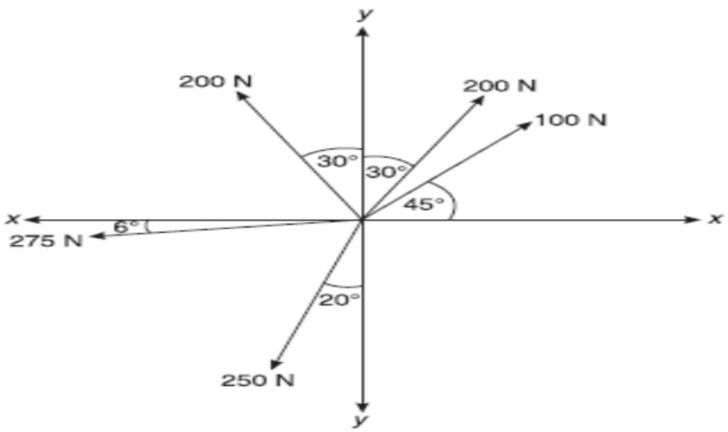
Find the resultant of the coplanar concurrent force system shown in Figure

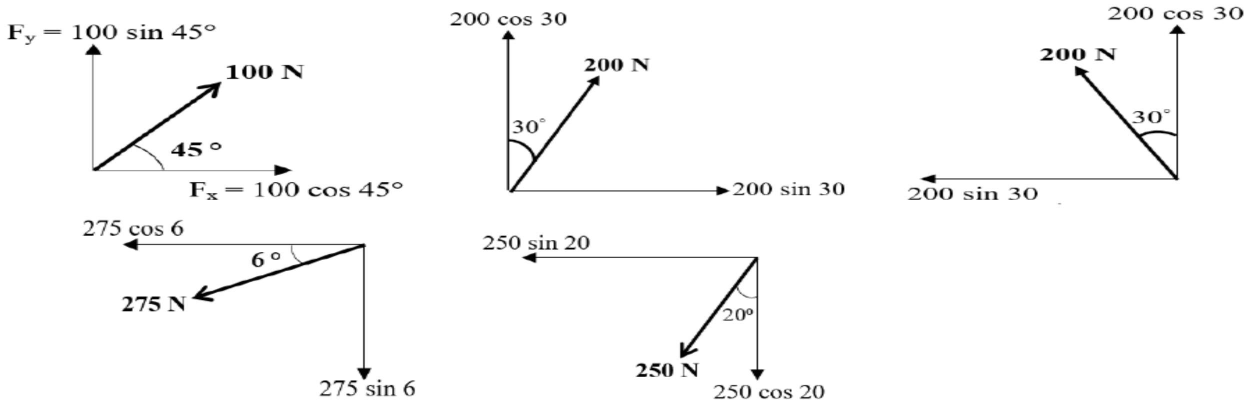






- Step 2: Summation of Forces along X and Y axis (□ Fx & □ Fy)
- □ Fx = 300 + 400 cos60 – 300 cos40
- □ Fx = 270.19 N
- □ Fy = 400 sin60 + 300 sin40 – 200
- □ Fy = 339.25 N
- Magnitude of Resultant force, R = √□Fx2+□Fy2
- R = √(270.19)2+ (339.25)2
- Magnitude of Resu ltant, R = 433.70 N
- Direction of Resultan t, □□x□□□tan−1 (□Fy
- □Fx)
- □□x = tan−1 (339.25
- 270.19)
- □□x = 51.47


y
200 N
200 N
100 N
30°
30°
45°
x
x
6°
275 N
20°
250 N
y


$F_y = 100 \sin 45°$
100 N
45 °
$F_x = 100 \cos 45°$
200 cos 30
200 N
30°
200 sin 30
200 cos 30
200 N
30°
200 sin 30
275 cos 6
6 °
275 N
275 sin 6
250 sin 20
20°
250 N
250 cos 20

- Step 2: Summation of Forces along X and Y axis (□ Fx & □ Fy)
- □ Fx = 100 cos45 + 200 sin30 – 200 sin30 – 275 cos 6 – 250 sin20
- □ Fx = – 288.29 N
- □ Fy = 100 sin45 + 200 cos30 + 200 cos30 – 275 sin 6 – 250 cos 20
- □ Fy = 153.45 N
- Step 3: Determination of Magnitude (R) and direction of Resultant (□□x)
- Magnitude of Resultant force, R = √□Fx2+□Fy2
- R = √(– 288.29)2+ (153.45)2
- Magnitude of Resultant, R = 326.59 N